DEPARTMENT OF MATHEMATICS

FINAL EXAM FOR FOURTH LEVEL (MATH STUDENTS)

TOTAL ASSESSMENT MARKS: 100 FINE ALLOWED: 2 HOURS TERM: FIRST

Answer the following questions:

(a) Find the general solution for the following linear difference equation:

$$\Delta f(x) + 4(1 - x^2)f(x) = 1 - 3x.$$

(20 marks)

(b) Discuss the stability for the following numerical procedure with respect to its initial values:

$$y_n - y_{n-3} = \frac{3h}{8} [f(x_{n-3}, y_{n-3}) + 3f(x_{n-2}, y_{n-2}) + 3f(x_{n-1}, y_{n-1}) + f(x_n, y_n)], n \ge 3.$$

(20 marks)

(II) (a) Apply the method of nets to compute the numerical solutions for the following mixed problem:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = \frac{x t}{2} , \quad u(x,0) = \ln(2-x) \quad (0 \le x \le 1);$$

$$u(0,t) = \ln 2, \ u(1,t) = 0 \quad (t \ge 0),$$

$$h = 0.05, \ \sigma = \frac{1}{2}.$$
(20 marks)

(b) Show how you can apply the method of finite differences to obtain the numerical solutions for the following boundary value problem:

$$y^{\parallel} + (1 + x^3)y^{\parallel} - x^2y = 0$$
, $x \in [1, 1.8]$; $y(1) = 0$, $y(1.8) = 1.96$, $h = 0.01$. (20 marks)

(III) (a) Define: An interior nodal point - The collocation points - Degenerate kernel - The Dirichlet problem.

(15 marks)

(b) Find an approximate solution for the following Fredholm's integral equation:

$$x(t) = t^3 + 1 + \int_0^1 (s^2 - 1) e^{-ts} x(s) ds$$
, $t \in [0,1]$.

(20 marks)

(IV) (a) Define ill and well-conditioned problems and whence study the stability of the following initial value problem with respect to its initial condition:

$$y'(x) = -[y(x)]^2$$
, $0 \le x \le b$; $y(0) = 1$.

(15 marks)

(b) Making use the method of least squares, construct an analytical approximate solution for the following boundary value problem:

$$y'' + xy' - y = 0$$
, $x \in [-1, 0]$; $y(-1) - y'(-1) = 3$, $y(0) + y'(0) = 0$.

(20 marks)

PROF. DR. A. R. M. EL-NAMOURY **EXAMINERS** DR. A. A. HEMEDA Y TIPL

Tanta University
Faculty of Science
Mathematics Department

Year: 4

Pages of questions: only one page







Date: 27/12/2017 Time allowed: 3 h Full Degree: 150

Subject: Topology 2 (MA4111)

Answer the following questions

Question 1:

- 1. Prove that in every T_0 space X, $\{x\}'$ is closed for every $x \in X$.
- 2. Discuss the compactness on real line \mathbb{R} with topologies: Discrete space- Indiscrete space- co-finite topology- usual topology.

Question 2:

- 1. Prove that every compact Hausdörff space is normal. (Is the converse is true?)
- 2. For a continuous mapping $f: (X, \tau_1, T_1) \to (Y, \tau_2)$. Prove that the inverse image of points in Y is closed set in X.

Question 3:

- 1. State with proof Hein-Borel Theorem.
- 2. If $X = \{a, b, c\}$ with a topology $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. Discuss the regularity, normality, T_3 and T_4 . (Draw the structure of space.)

Question 4:

- 1. For every topological space (X, τ) and $A \subset X$, prove that A is compact with respect to τ iff A is compact with respect to τ_A .
- 2. Prove that the usual topology is Hausdörff.

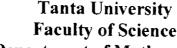
Question 5:

- 1. Prove that a continuous one-to-one function from a compact space onto a Hausdörff space is homeomorphism.
- 2. Show that every subset from a co-finite topology is compact.

With best wishes

Dr. Abd El-Fattah El-Atik

FACULTY OF SCIENCE - TU



Department of Mathematics

Final Exam for the First Semester 2017-2018 Course title: Differential geometry(1) Course Code: MA4107

Date: 30 /12 /2017 Total mark: 150 Marks

Time allowed: 2 Hours

Answer all the following questions:

First question: Choose the correct answer (30 Marks):

1-If $\alpha(t) = (x(t), y(t))$ is a positively – oriented simple closed curve in \mathbb{R}^2 With period a, then A equal

(a)
$$A = \frac{1}{2} \int_0^a (xy - yx) dt$$
.

(b)
$$A = \frac{1}{2} \int_0^a (xy - yx) dt$$

(c)
$$A = \frac{1}{2} \int_0^a (xy + yx) dt$$
.

2-If $\alpha(s)$ is a unit speed curve of R^2 , then

(a)
$$\alpha(s) = \alpha'(s)$$

(b)
$$\alpha(s) = -\alpha'(s)$$

(b)
$$\alpha(s) = -\alpha'(s)$$
 (c) $\alpha'(s)$ is orthogonal to $\alpha''(s)$

3-Let lpha be a simple closed curve, let ℓ be its length and let A be the area. Then

(a)
$$A > \frac{\ell^2}{4\pi}$$
 (b) $A < \frac{\ell^2}{4\pi}$ (c) $A \le \frac{\ell^2}{4\pi}$

$$(b) A < \frac{\ell^2}{4\pi}$$

$$(c) A \leq \frac{\ell^2}{4\pi}$$

4-Every convex simple closed curve in \mathbb{R}^2 has at least

- (a) Four vertices
- (b) Two vertices
- (c)Three vertices

5-The parameterization of the curve $\chi_3^2 = 1 - \chi_1$, $\chi_2^2 = \chi_1$

(a)
$$\alpha(t) = (\cos t, \sin t)$$
 (b) $\alpha(t) = (\cos^2 t, \cos t, \sin t)$ (c) $\alpha(t) = (\sec t, \sinh t)$

6- A vertex of a curve $\alpha(s)$ in \mathbb{R}^2 is a point where

(a)
$$k > 0$$

(a)
$$k > 0$$
 (b) $k < 0$ (c) $k = 0$

$$(c)$$
 $\mathbf{k} = 0$

7-The parameterization of the curve $\frac{x^2}{2^2} + \frac{y^2}{2^2} = l$ equal

(a)
$$\alpha(t) = (3\cos t, 2\sin t)$$
 (b) $\alpha(t) = (2\cos t, 3\sin t)$ (c) $\alpha(t) = (\sec t, \sinh t)$

8-Which one of the following statements is true

9-The equation of tangent line of the curve $\alpha(s)$ equal

(a)
$$\vec{r} = \alpha(s_0) + \lambda T$$
 (b) $\vec{r} = \alpha(s_0) + \lambda N$ (c) $\vec{r} = \alpha(s_0) + \lambda B$

(Please Turn the Paper)





TANTA UNIVERSITY FACULTY OF SCIENCE DEPARTMENT OF MATHEMATICS

EXAMINATION FOR PROSPECTIVE STUDENTS (4TH YEAR) STUDENTS OFMATHEMATICS

COURSE TITLE:GENERAL RELATIVITY

COURSE CODE:MA4113

Answer fife questions:

- [1] (a) Prove that $ds^2 = dx^2 + dy^2 + dz^2$ in parabolic coordinates (ξ, η, φ) given as $ds^2 = (\xi^2 + \eta^2)d\xi^2 + (\xi^2 + \eta^2)d\eta^2 + \eta\xi d\varphi^2$ where $x = \xi\eta\cos\varphi$, $y = \xi\eta\sin\varphi$, $z = (\xi^2 \eta^2)/2$.
 - (b) Prove that the Christoffel's symbol of second kind is not tensor.
- [2] (a) The equation $K(ij)A_{jk} = B_{ik}$ holds for all the coordinates systems. If A and B are second-rank tensors, show that K is a second tensor also.
 - (b) Use the question [3] (b) and Obtain the equations of the geodesic for the same metric
 - (c) derive the Euler equations.
- [3] (a) Prove that $B_{i;j}$ is tensor of rank two and find Riemann-Christoffel's tensor $B_{\mu\nu\sigma}^{\rho}$.
 - (b) Find the fundamental metric and Christoffel's symbols to the metric

$$ds^{2} = dt^{2} - e^{2kt} \left(dx^{2} + dy^{2} + dz^{2} \right)$$

Dr. Mohamed Khalifa

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EXAMINERS

DR./MOHAMED ABDOU KHALIFA	PROF./OMEAR SHAKER
PROF/SLIM ALLMOHAMMADEIN	DR/

With my best wishes





Tanta University Faculty of Science

Department of Mathematics

Final term exam for the first semester 2017-2018

Course title: Operations Research (2)

Date: 6 /1 /2018 Total Marks: 150

Course code: MA4105

Time allowed: Hours

Answer all the following questions:

First question: (40 Marks)

(a) Discuss the convexity of the following sets:

(i)
$$S = \{x : x = (x_1, x_2) : x_1 \ge 2, x_2 \le 4 \}$$
.

(ii)
$$S = \{x : g_i(x) \le 0, i = 1, 2, \dots, m\}$$
, where $g_i(x)$ are convex functions.

(b) Find whether the $f(x) = 25x_1^2 + 34x_2^2 + 41x_3^2 - 24x_2x_3$ is a positive definite or not.

(c) Prove that if f(x) and g(x) be convex functions defined over a convex set S then the sum f(x)+g(x) is a convex function?.

Second question: (40 Marks)

(a) Find the local extrema $f(x_1, x_2, x_3) = x_1^2 + (x_1 + x_2)^2 + (x_1 + x_3)^2$.

(b) Using Lagrangian multipliers to find the extreme points for the function

$$z = f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

Subject to the constraint: $x_1 + 2x_2 = 2$, $x_1, x_2 \ge 0$

Determine whether the etreme points are maximum or minimum.

Third question: (30 Marks) ok

(a) Minimize $f(x) = x_1^2 + x_2^2 + 2x_1 + 4x_2 + 5$ using the steepest descent method starting at the point $x_1 = 0$ and $x_2 = 0$.

(b) By direct substitution method solve the following NLPP

$$\min f(x) = x_1^2 + (x_2 + 1)^2 + (x_3 - 1)^2$$
 subject to constraint $x_1 + 5x_2 - 3x_3 = 6$.

Fourth question: (40 Marks)

(a) Show that the following function is convex

$$f(x_1, x_2, x_3) = 3x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3 - 6x_1 - 4x_2 - 2x_3$$

(b) Use the Kuhn-Tucker conditions to solve the NLPP:

maximize
$$z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$$
 subject to $2x_1 + x_2 \le 5$, $x_1, x_2 \ge 0$.

(Best wishes)

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TANTA UNIVERSITY FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

FINAL EXAM FOR FRESHMEN (LEVEL 4) STUDENTS OF COMPUTER SCIENCES

FINAL EXAM FOR PRESHIVEN (LEVEL 4) STUDENTS OF COMPUTER SCIENCES.						
1956	COURSE TITLE:	OPERATIONS RE	SEARCH	COURSE CODE:MA4105		
DATE:	JAN. 6, 2018	TERM: FIRST	TOTAL ASSESSMENT MARKS:150	TIME ALLOWED:TWO HOURS		

Answer on the following questions:

Question 1:(30 marks)

Discuss the feasibility of the problem which has the following simplex tablue

	-z	x	У	l U ₁	U ₂	d	
-z	1	0	-2	0	0	5	
-w	0	2	-3	1	1	5	
U_1	0	-1	1	1	0	3	
U ₂	0	-1	2	0	1	2	

Question 2:(40 marks)

Determine the optimal solution from the following simplex tablue

	-z	х	У	Si	S ₂	d	
-z	1	0	-2	0	0	5	
х	0	2	1	1	1	3	
S ₂	0	0	2	0	1	2	

Question 3: (40 marks)

Find the solution of dual without making duality for the following program

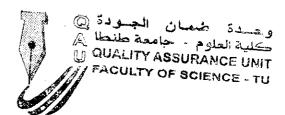
 $\begin{aligned} \max & \mathsf{Z} = \mathsf{X}_1 + \! 2 \mathsf{X}_2 \\ & \mathsf{Subject to} \\ & \mathsf{X}_1 + \! \mathsf{X}_2 \leq 20, \\ & 2 \mathsf{X}_1 + \! 3 \mathsf{X}_2 \leq \! 60, \end{aligned}$

 X_1 , $X_2 \ge 0$

Question 4: (40 marks)

Put the following system in its canonical form

x+y+z=5, 2x-y-z=3



EXAMINERS	E A. Youne S	PROF. EBRAHIM YOUNESS
!	E.D. Duncas	PROF. E. A. AMMAR